

# Approximation of Green's Functions

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Special symmetries of the Green's functions of a non-relativistic many fermion-system and conservation laws, expressible by hermitian generators, are formulated as relations for a Green's function operator. Approximations for the Green's functions, defined as partial summations of the perturbation expansion, and consistent with the symmetries and conservation laws are presented.

## Introduction

This paper is concerned with the Green's function

$$\langle \Psi_0 | \tau \{ A^+ (0) B(t) \} | \Psi_0 \rangle,$$

especially with

$$\langle \Psi_0^* | \tau \{ a_1^+ (0) a_2 (0) a_3^+ (t) a_4 (t) \} | \Psi_0 \rangle.$$

There  $\tau$  stands for the time ordered product of the operators. This function gives information about the excitation energies of fermion systems. An approximation often used for this function is the Random-Phase-Approximation (RPA). We will give approximations which are of higher order than the RPA and which are consistent with the conservation laws of the Hamiltonian. For this purpose it is necessary to choose a concept of reduction of graphs which does not agree with the usual concept of reduction used for Green's functions of this type<sup>1</sup>. We do not distinguish between self-energy-graphs and other ones. The method developed here has been applied to negative parity excited states of He<sup>4</sup> and in the case of the quasi spin-model of Lipkin *et al.*<sup>3,2</sup> as a test example.

## 1. Definition of the Green's Functions

The Hamiltonian of our fermion-system is

$$H = H_0 + \lambda V_1. \quad (1)$$

1 Thouless, D. J.: The quantum mechanics of many-body-systems. New York-London: Academic Press 1961.

2 Fuchssteiner, B.: Partialsummationen für die Zweiteilchen-Greenfunktionen im Zusammenhang mit der Störungsrechnung. Institut für Theoretische Kernphysik der Technischen Hochschule Darmstadt (1965), unpublished.

3 Lipkin, H. J., Meshkov, N., Glick, A. J.: Validity of many-body-approximation-methods for a solvable model. Nucl. Phys. 62, 188—224 (1965).

$H_0$  is the unperturbed Hamiltonian,  $\lambda V_1$  is the perturbation. Later on the Green's functions will be represented by the perturbation series with respect to  $\lambda V_1$ , i.e. by a power expansion in  $\lambda$ . In the space of one-particle-states a basis is chosen in a way that  $H_0$  and  $V_1$  are given by

$$H_0 = \sum_i H_i a_i^+ a_i, \tag{2}$$

$$V = 1/2 \sum_{i,j,k,l} V_{i,j,k,l} a_j^+ a_i^+ a_k a_l. \tag{3}$$

The Green's functions under consideration are defined by

$$G_{AB}(t) = \langle \Psi_0 | \tau \{ A^+(0) B(t) \} | \Psi_0 \rangle \tag{4}$$

where  $|\Psi_0\rangle$  is the true ground state of the  $n$  fermion system.

$$H |\Psi_0\rangle = |\Psi_0\rangle E_0. \tag{5}$$

If  $[A^+, N] = mA^+$  and  $[B, N] = -mB$  then the time ordered product is defined by

$$\tau \{ A^+(0) B(t) \} = \begin{cases} A^+(0) B(t) & \text{if } t < 0 \\ (-1)^m B(t) A^+(0) & \text{if } t \geq 0. \end{cases} \tag{6}$$

$N$  is the operator of the particle number.

$$B(t) = \exp(iHt) B(0) \exp(-iHt). \tag{7}$$

$A^+(0)$  and  $B(0)$  are fixed operators, independent of time, and will be called  $A^+, B$  in the following.

Sometimes we will deal with the Fourier-transform of  $G_{AB}(t)$

$$\begin{aligned} G_{AB}(z) &= i \int_{-\infty}^{+\infty} G_{AB}(t) e^{-izt} dt \\ &= \langle \Psi_0 | A^+ (H - E_0 - z)^{-1} B | \Psi_0 \rangle \\ &\quad + \langle \Psi_0 | B (H - E_0 + z)^{-1} A^+ | \Psi_0 \rangle. \end{aligned} \tag{8}$$

From (8) we see that the poles of  $G_{AB}(z)$  in the  $z$ -plane determine excitation energies of the fermion system relative to the ground state of the  $n$  particle system. We assume that the number of states is finite, thus we avoid difficulties with respect to the analytical behaviour of the function  $G_{AB}(z)$  in the  $z$ -plane;  $G(z)$  is then a meromorphic function and (8) holds.

By complex conjugation of (8) one obtains

$$G_{AB}(z) = G_{BA}^*(z^*). \tag{9}$$

Substituting  $z$  by  $-z$  results in

$$G_{AB}(z) = G_{B^+ A^+}(-z). \quad (10)$$

If the ground state  $|\Psi_0\rangle$  is an eigenstate of the hermitian operator  $T$  and if the conservation law

$$[H, T] = HT - TH = 0$$

holds, one gets Eqs. (11) and (12). We only consider conservation laws with  $[T, N] = 0$ , where  $N$  is the operator of the particle number.

$$\begin{aligned} \langle \Psi_0 | [T, A^+] (H - E_0 - z)^{-1} B | \Psi_0 \rangle \\ = \langle \Psi_0 | A^+ (H - E_0 - z)^{-1} [B, T] | \Psi_0 \rangle, \end{aligned} \quad (11)$$

$$\begin{aligned} \langle \Psi_0 | [T, B] (H - E_0 + z)^{-1} A^+ | \Psi_0 \rangle \\ = \langle \Psi_0 | B (H - E_0 + z)^{-1} [A^+, T] | \Psi_0 \rangle. \end{aligned} \quad (12)$$

Or, using the definition of  $G_{AB}(z)$

$$G_{[A, T]B}(z) = G_{A[B, T]}(z). \quad (13)$$

The Eqs. (9), (10) and (13) are called, resp., the symmetries (9), (10) and the conservation laws (13) of the Green's function\*.

For a simpler formulation of the properties of the Green's function a vector space  $\mathfrak{Q}$  is introduced.  $\mathfrak{Q}$  is the space of all linear operators over the states of the fermion system; i.e. the operators  $H, H_0, T$  etc. are elements of  $\mathfrak{Q}$ . We consider a mapping

$$\mathfrak{Q} \xrightarrow{\varphi} \mathfrak{Q}(\mathfrak{Q}, \mathfrak{Q})$$

i.e. a mapping of the elements of  $\mathfrak{Q}$  into the operators over  $\mathfrak{Q}$ .

$$A \in \mathfrak{Q}, \quad B \in \mathfrak{Q}, \quad \varphi(A) | B \rangle \stackrel{\text{def}}{=} | [A, B] \rangle. \quad (14)$$

We want to introduce in  $\mathfrak{Q}$  a positive definite metric. Again we use the notation of second quantisation.

We define

$$\begin{aligned} (A | B) \stackrel{\text{def}}{=} \langle 0 | A^+ B | 0 \rangle \\ + 1/1! \sum_i \langle i | \{ A^+ B - a_i^+ A^+ B a_i \} | i \rangle \\ + 1/2! \sum_{i_1 i_2} \langle i_1 i_2 | \{ A^+ B - a_{i_1}^+ A^+ B a_{i_1} - a_{i_2}^+ A^+ B a_{i_2} \} | i_1 i_2 \rangle \end{aligned}$$

\* Since we have assumed that the number of states is finite we do not need an investigation either of the existence of  $[A, T]$  or of the convergence properties of the sum in (15).

$$\begin{aligned}
 &+ a_{i_2}^+ a_{i_1}^+ A^+ B a_{i_1} a_{i_2} \} |i_1 i_2\rangle \\
 &+ \dots + 1/n! \sum_{i_1 \dots i_n} \langle i_1 \dots i_n | \{ A^+ B - \sum_{k=1}^n a_{i_k}^+ A^+ B a_{i_k} \\
 &+ \sum_{k,r=1}^n (a_{i_k} a_{i_r})^+ A^+ B a_{i_k} a_{i_r} \\
 &+ \dots (-1)^n (a_{i_1} \dots a_{i_n})^+ A^+ B a_{i_1} \dots a_{i_n} \} |i_1 \dots i_n\rangle \\
 &+ \dots
 \end{aligned} \tag{15}$$

$|0\rangle$  is the vacuum state and  $|i_1 \dots i_n\rangle$  is  $(a_{i_1}^+ a_{i_2}^+ \dots a_{i_n}^+) |0\rangle$ .

If  $A, B$  are operators of the form  $a_i^+ a_k$  we have

$$(A|B) = \sum_i \langle i | A^+ B | i \rangle.$$

If  $A, B$  are operators of the form  $a_i^+ a_k^+ a_r a_s$  we have

$$(A|B) = 1/2 \sum_{ik} \langle ik | A^+ B | ik \rangle \quad \text{etc.}$$

That means if  $A, B$  are operators which propagate  $n$  holes and then  $n$  particles we have to take for  $(A|B)$  the trace of  $A^+ B$  in the subspace of the  $n$  fermion states. The form of the terms of higher order in (15) is chosen in a way that the trace in subspaces of more than  $n$  particles does not contribute to  $(A|B)$ . We denote with  $T_r^{(n)}$  the trace in the subspace of the  $n$  fermion system.

Now, if  $\varphi(N)|A\rangle=0$  it follows in the usual way

$$T_r^{(n)}(AB) = T_r^{(n)}(BA). \tag{16}$$

A conjugation of an operator over  $\mathfrak{Q}$  is denoted by a tilde

$$(A|G|B) = (B|\tilde{G}|A)^*. \tag{17}$$

If  $T$  is an hermitian operator and if  $\varphi(N)|T\rangle=0$  does hold, then  $\varphi(T)$  is also hermitian, i.e.  $\widetilde{\varphi(T)} = \varphi(T)$ . To prove this statement we only have to show that  $T_r^{(n)}(A^+ [T, B]) = T_r^{(n)}([A^+, T] B)$  holds, because we have in (15) a sum over traces in subspaces with fixed particle numbers.

$$T_r^{(n)}(A^+ [T, B]) - T_r^{(n)}([A^+, T] B) = T_r^{(n)}(TA^+ B) - T_r^{(n)}(A^+ BT).$$

Now applying (16) we prove the statement. With the Green's function there corresponds an operator over  $\mathfrak{Q}$  which is given by its matrix elements

$$(B|G(z)|A) \stackrel{\text{def}}{=} G_{BA}(z), \tag{18a}$$

$$(B|G(t)|A) \stackrel{\text{def}}{=} G_{BA}(t). \tag{18b}$$

A conjugation  $K$  is defined in  $\mathfrak{Q}$  by

$$K|A) = |A^+).$$

Now we can formulate (9), (10) and (13) in another way

$$G(z) = \widetilde{G}(z^*), \quad (19a)$$

$$G(-z)K = KG(z^*), \quad (19b)$$

$$[G(z), \varphi(T)] = 0. \quad (19c)$$

With

$$G(t) = (1/2\pi i) \int_{-\infty}^{+\infty} G(z) e^{itz} dz$$

we get the corresponding relations for  $G(t)$

$$\widetilde{G}(t) = -G(-t^*), \quad (20a)$$

$$KG(t) = -G(t^*)K, \quad (20b)$$

$$[G(t), \varphi(T)] = 0. \quad (20c)$$

In the following chapter we will consider approximations for the Green's function. Of course it will be desirable to get approximations which are consistent with the symmetries and conservation laws, i.e. the approximation for the Green's function should not disturb the conservation laws, given by the Hamiltonian, and the symmetries, given by the structure of the Green's function. We therefore give the following

*Definition.* An approximation is consistent with the symmetries and the conservation law  $[H, T] = 0$  if Eqs. (19) and (20) resp. do hold for the approximated Green's function.

It is sometimes convenient to investigate the operator for the Green's function in a subspace  $P\mathfrak{Q}$  of  $\mathfrak{Q}$ ,  $P\mathfrak{Q} \subset \mathfrak{Q}$ , where  $P$  is a projection operator.

$$P^2 = P, \quad P = \widetilde{P}.$$

Then we have

$$G^P(z) = PG(z)P$$

$$G^P(t) = PG(t)P.$$

For checking the approximations with regard to the relations (19) and (20), resp., it is desirable to choose a projection with the properties

$$[\varphi(T), P] = 0, \quad (21a)$$

$$PK = KP. \quad (21b)$$

Then the operator  $G^P$  satisfies (19) and (20), resp. If (21a) does not hold we must proceed in a different way. In this case we decompose the space  $\mathfrak{Q}$  of the operators over the fermion states with respect to different eigenvalues of the hermitian operator  $\varphi(T)$

$$\mathfrak{Q} = \mathfrak{Q}_0 \oplus \mathfrak{Q}_1 \oplus \dots = \sum_i \mathfrak{Q}_i. \tag{22}$$

$\mathfrak{Q}_i$  is the space of all states belonging to the same eigenvalue  $\varphi_i$  of  $\varphi(T)$ , i. e. if  $|\chi\rangle \in \mathfrak{Q}_i$  we have

$$\varphi(T) |\chi\rangle = |\chi\rangle \varphi_i.$$

This decomposition is unique since we have assumed a finite number of states and a hermitian  $T$ . Now, if  $\Pi_m$  denotes the projection operator onto  $\mathfrak{Q}_m$  then by virtue of (19c) we have

$$\begin{aligned} [G(z), \Pi_m] &= 0 \\ [G(t), \Pi_m] &= 0. \end{aligned} \tag{23}$$

If

$$[P, \Pi_m] = 0 \tag{24}$$

then (23) also holds for  $G^P$ , which of course is a weaker condition than (19c). In this case (23) should also hold for consistent approximations. This procedure plays a role in nuclear shell model calculations with respects to spurious excitations of the centre of mass-motion.

### 2. Perturbation Expansions for Green's Functions

Let us consider the particle-hole Green's function. We assume that we have a ground state  $|\phi_0\rangle$  of the unperturbed Hamiltonian which is a Slater determinant. The particle-hole Green's functions is defined by the matrix elements of  $G(t)$

$$(A | G(t) | B)$$

where  $A$  and  $B$  are linear forms of

$$a_i^\dagger \ a_k$$

where  $i$  is a hole and  $k$  is a particle or where  $i$  is a particle and  $k$  is a hole.

$i$  is defined to be a hole state only if  $a_i^\dagger |\phi_0\rangle = 0$ , correspondingly  $k$  is a particle state only if  $a_k |\phi_0\rangle = 0$ .  $G_0(t)$  is the operator for the unperturbed Green's function, i.e.

$$(B | G_0(t) | A) = \begin{cases} \langle \phi_0 | A^\dagger \exp(iH_0 t - iW_0 t) B | \phi_0 \rangle & t < 0 \\ \langle \phi_0 | B \exp(iW_0 t - iH_0 t) A^\dagger | \phi_0 \rangle & t \geq 0 \end{cases}$$

where

$$H_0 |\phi_0\rangle = |\phi_0\rangle W_0$$

$$G_0(z) = i \int_{-\infty}^{+\infty} \exp(-izt) G_0(t) dt.$$

Sometimes it will be convenient to consider a subset of all particle-hole Green's functions. Therefore we assume that a projection operator  $P$  onto a subspace of the particle-hole operators is given, with

$$[K, P] = 0 \tag{25}$$

i.e. if  $a_i^+ a_k \in P\mathcal{Q}$  then it follows that  $a_k^+ a_i \in P\mathcal{Q}$ .

From the definition (15) of the metric it follows

$$(a_i^+ a_k | a_r^+ a_s) = \delta_{ir} \delta_{ks}.$$

For the unit operator over  $P\mathcal{Q}$  there holds

$$1 = \sum_{ik \in P\mathcal{Q}} |a_i^+ a_k\rangle (a_i^+ a_k|).$$

In the following we shall employ the usual graphical notation. In order to facilitate the notation we give a brief summary of the rules for the calculation with graphs. A detailed description of these rules can be found in<sup>1</sup>.

All graphs linked with the external lines in Fig. 1 contribute to the Green's function  $(a_a^+ a_b | G(t) | a_c^+ a_d)$  according to the rules  $t1-t4$ .

Sometimes the external lines are representative for an external interaction but here they stand only to denote the beginning and the end of a graph. The points at the external lines are called junctions. With  $\sigma_1, \sigma_2 \dots$  we denote the time variables of the interaction lines which are between  $\sigma=0$  and  $\sigma=t$ ,  $\theta_1, \theta_2 \dots$  are the time variables for which we have  $\theta_i < 0, \theta_i < t$ , and for  $\tau_1, \tau_2 \dots$  we have  $\tau_k > 0, \tau_k > t$ .

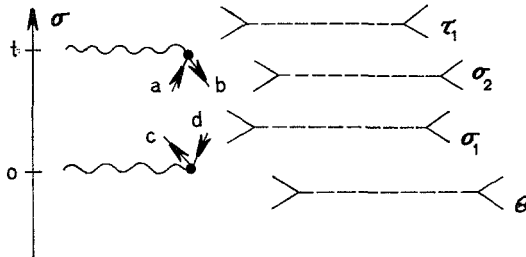


Fig. 1. A graph with external lines

Rule t1. Each interaction line (Fig. 2) gives a matrix element  $(\lambda/2i)V_{abcd}$ .



Fig. 2. An interaction line

Rule t2. Each closed loop and each hole line gives a factor  $-1$ .

Rule t3. A hole line  $l$  in the interval  $\Delta\sigma$  gives a factor  $\exp(i\Delta\sigma H_l)$  and a particle line  $k$  gives a factor  $\exp(-i\Delta\sigma H_k)$ .

Rule t4. We must carry out an overall integration

$$\int_t^\infty d\tau_k \dots \int_t^{\tau_2} d\tau_1 \int_0^t d\sigma_n \dots \int_0^{\sigma_2} d\sigma_1 \int_{-\infty}^0 d\theta_n \dots \int_{-\infty}^{\theta_2} d\theta_1.$$

In order to describe the "reduction of graphs" we define the following notions.

*Definition 1:* The exchange of a pair of particle-hole-lines is a cut of the lines at the point  $\sigma = \sigma_0$  followed by a connection of different lines resulting in junctions at the point  $\sigma = \sigma_0$  (Fig. 3).

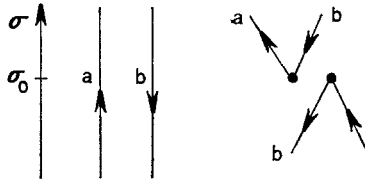


Fig. 3. The exchange of two lines

To each pair of particle-hole-lines corresponds a one-particle operator which generates the given pair, i.e. the lines in Fig. 2 are generated by  $a_a^+ a_b$ .

*Definition 2.* By distinction of pairs we understand a distinction of all pairs of lines for which the following conditions are fulfilled

- a) Both lines run through the same time interval  $\Delta\sigma$ .
- b) The lines must not end up in a junction which connects them both.
- c) After the exchange of a pair there is no connected part of a graph with more than two junctions.
- d) To each pair there corresponds an operator  $\in P\Omega$ .

e) The exchange of a pair converts a connected part of the graph into a disconnected one.

*Definition 3.* A reduction of a graph is the exchange of a distinguished pair.

*Definition 4.* A reduction of the first kind leaves all distinguished pairs distinguished except for the exchanged pair.

*Definition 5.* A first kind irreducible graph is a graph which cannot be reduced by a reduction of the first kind.

The difference between the notions “reduction” and “reduction of the first kind” is necessary because it may be possible that a first kind irreducible graph can be reduced by a reduction. In Fig. 4 an example is given for a first kind irreducible graph which can be reduced by the exchange of  $(ac)$  or by exchange of  $(bd)$ .

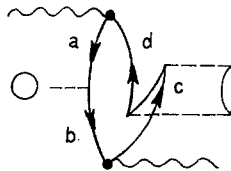


Fig. 4. An irreduzible graph

*Definition 6.* A first kind irreducible graph belongs to the class  $[\gamma_n]$  if exactly  $n$  different decompositions in first kind irreducible graphs are possible.

A decomposition is an arbitrary number of reductions. Two decompositions are equal if the set of reduced pairs is equal.

In the following we denote the first kind irreducible graphs as “irreducible graphs”. Fig. 5 shows an irreducible graph  $\in [\gamma_2]$ . Decompositions are possible by reducing the following pairs. 1.  $(ad)$  2.  $(ab)$  and  $(ac)$ . The reduction of  $(ab)$  or  $(ac)$  resp. leads to graphs which are not irreducible.

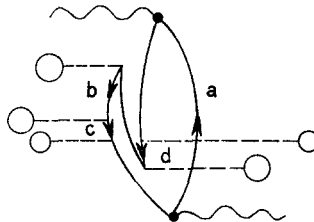


Fig. 5. A graph  $\in [\gamma_2]$

*Definition 7.* A vertex term  $\gamma_{cd,ba}^{(n)}(t_1 - t_2)$  (Fig. 6a) and  $\bar{\gamma}_{cd,ba}^{(n)}(t_1 - t_2)$  (Fig. 6b) resp. is an irreducible graph  $\in [\gamma_n]$  where we have omitted the junction at the lines  $(ab)$  (Fig. 6a) and  $(cd)$  (Fig. 6b) resp. The contribution of a vertex term differs from the contribution of the corresponding irreducible graph because we leave out the integration over the time of the last interaction line ( $\sigma = t_2$ ) at the pair  $(ab)$  and  $(cd)$  resp. The pair  $(ab)$  and  $(cd)$  resp. does only count as far as to the point  $\sigma = t_2$ .

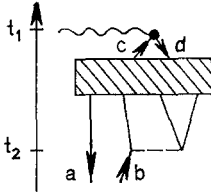


Fig. 6a. A vertex contributing to  $\gamma_{cd,ba}^{(n)}(t_1 - t_2)$

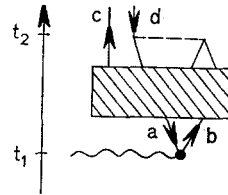


Fig. 6b. A vertex contributing to  $\bar{\gamma}_{cd,ba}^{(n)}(t_1 - t_2)$

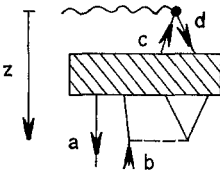


Fig. 7a. A vertex contributing to  $\gamma_{cd,ba}^{(n)}(z)$

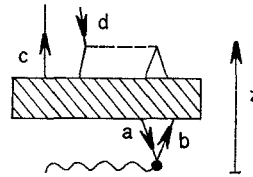


Fig. 7b. A vertex contributing to  $\bar{\gamma}_{cd,ba}^{(n)}(z)$

This notion is useful because the convolution of vertex terms with graphs contributing to  $G(t)$  gives again graphs contributing to  $G(t)$ . Again we define the corresponding operators

$$\begin{aligned} (a_c^+ a_d | \gamma^{(n)}(t) | a_b^+ a_a) &= \gamma_{cd,ba}^{(n)}(t) \\ (a_c^+ a_d | \bar{\gamma}^{(n)}(t) | a_b^+ a_a) &= \bar{\gamma}_{cd,ba}^{(n)}(t). \end{aligned} \tag{26}$$

Let us consider now the following two special convolutions

$$\begin{aligned} G_0(t) + \sum_{k=1}^{\infty} \int_{-\infty}^{+\infty} \Gamma(t - \sigma_1) \Gamma(\sigma_1 - \sigma_2) \dots \Gamma(\sigma_{k-1} - \sigma_k) \\ \cdot G_0(\sigma_k) d\sigma_1 \dots d\sigma_k, \end{aligned} \tag{27a}$$

$$\begin{aligned} G_0(t) + \sum_{k=1}^{\infty} \int_{-\infty}^{+\infty} G_0(t - \sigma_1) \bar{\Gamma}(\sigma_1 - \sigma_2) \dots \bar{\Gamma}(\sigma_{k-1} - \sigma_k) \\ \cdot \bar{\Gamma}(\sigma_k) d\sigma_1 \dots d\sigma_k \end{aligned} \tag{27b}$$

where

$$\Gamma(t) = \sum_{n=0}^{\infty} (1-n) \gamma^{(n)}(t)$$

$$\bar{\Gamma}(t) = \sum_{n=0}^{\infty} (1-n) \bar{\gamma}^{(n)}(t).$$
(27c)

We shall see that the sum of integrals (27a) and (27b) resp. is equal to the sum of all graphs contributing to  $G(t)$ , i.e.

$$G(t) = G_0(t) + \sum_{k=1}^{\infty} \int_{-\infty}^{+\infty} \Gamma(t-\sigma_1) \dots G_0(\sigma_k) d\sigma_1 \dots d\sigma_k,$$
(28a)

$$G(t) = G_0(t) + \sum_{k=1}^{\infty} \int_{-\infty}^{+\infty} G_0(t-\sigma_1) \dots \bar{\Gamma}(\sigma_k) d\sigma_1 \dots d\sigma_k.$$
(28b)

To prove this statement it is only necessary to look at the irreducible graphs because the reduction of the first kind is unique. We assume that each irreducible graph which belongs to the classes  $[\gamma_0] \dots [\gamma_N]$  is contained only once in (27a) and (27b) resp. In all convolutions of the graphs  $[\gamma_k]$ ,  $k < N+1$ , then the graphs  $\in [\gamma_{N+1}]$  are contained  $(N+1)$  times, this follows from the definition of  $[\gamma_{N+1}]$ . Convoluting the term of order  $N+1$  of  $\Gamma(t)$ ,  $\bar{\Gamma}(t)$  in (27c) i.e.  $-N\gamma^{(N+1)}(t)$  and  $-N\bar{\gamma}^{(N+1)}(t)$  resp. with  $G_0(t)$  we obtain the graphs  $\in [\gamma_{N+1}]$   $(-N)$  times. The graphs  $\in [\gamma_{N+1}]$  contribute only once to the sum of integrals (27).

Because our assumption is true for  $N=0$  it is also true for all  $N$ , which proves our statement.

Similar arguments are frequently used if integral equations for Green's functions are derived<sup>1</sup>. However the fact that in general the reduction is not unique has never been considered. We see that (28) are the Neumann series of the following integral Eqs. (29). From now on we shall consider only these integral equations.

$$G(t) = G_0(t) + \int_{-\infty}^{+\infty} \Gamma(t-\sigma) G(\sigma) d\sigma,$$
(29a)

$$G(t) = G_0(t) + \int_{-\infty}^{+\infty} G(t-\sigma) \bar{\Gamma}(\sigma) d\sigma.$$
(29b)

In order to simplify the calculation we consider the Fourier transform

$$G(z) = G_0(z) + \Gamma(z) G(z)$$
(30a)

$$G(z) = G_0(z) + G(z) \bar{\Gamma}(z)$$
(30b)

where

$$\Gamma(z) = \int_{-\infty}^{+\infty} \Gamma(t) \exp(-izt) dt$$

$$\bar{\Gamma}(z) = \int_{-\infty}^{+\infty} \bar{\Gamma}(t) \exp(-izt) dt.$$

Next we consider approximations for  $G(z)$  which are given by approximations for  $\Gamma(z)$  and  $\bar{\Gamma}(z)$  resp. and Eq. (30). For this purpose it is necessary to formulate the Eqs. (19) and (23) with respect to  $\Gamma, \bar{\Gamma}$ .

Using (19) and (30) we obtain

$$\Gamma(z) = \tilde{\Gamma}(z^*), \tag{31a}$$

$$K\Gamma(z) = \Gamma(-z^*)K, \tag{31b}$$

$$[\Gamma(z), \varphi(T)] = 0 \tag{31c}$$

$$[\bar{\Gamma}(z), \varphi(T)] = 0.$$

From (24) and (30) we obtain

$$[\Gamma(z), \Pi_m] = 0 \tag{32}$$

$$[\bar{\Gamma}(z), \Pi_m] = 0.$$

Of course we get consistent approximations for  $G(z)$  if we take consistent approximations for  $\Gamma, \bar{\Gamma}$  i.e. approximations which fulfill (31 a, b) and (31 c) and (32) resp. The only question is how to get consistent approximations for  $\Gamma, \bar{\Gamma}$ . For this purpose we take the Taylor series for  $\Gamma$  and  $\bar{\Gamma}$  resp.

$$\Gamma(z) = \sum_1^{\infty} \lambda^n \delta_n(z)$$

$$\bar{\Gamma}(z) = \sum_1^{\infty} \lambda^n \bar{\delta}_n(z). \tag{33}$$

If this series has a radius of convergence greater than zero the Eqs. (33) are identities in  $\lambda$ . Then (31) and (32) are valid for any  $\delta_n$  and  $\bar{\delta}_n$  resp. Now, if we approximate  $\Gamma, \bar{\Gamma}$  by any finite sum of  $\delta_n, \bar{\delta}_n$  then Eqs. (31) and (32) hold for the approximation as well. Eq. (30) represents then a consistent approximation for  $G(z)$ . In this sense the RPA is a consistent approximation. If we consider approximations of higher order than the RPA, we see that one can not distinguish between self-energy-graphs and other ones.

Furthermore if one uses integral equations which are derived without considering the difference between 'reduction' and 'reduction of the first kind' the graphs of  $\gamma_n (n > 1)$  are counted in a wrong way. However this occurs only in approximations of order higher than 2.

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